Doppler Institute - CRM Workshop

on the occasion of 80th birthdays of
Jiří Patera and Pavel Winternitz

May 30 - June 3, 2016

Organizing Committee

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Monday 30.5. CTU B103

9:30 – 09:45	Opening
9:45 – 10:35	**Luc Vinet** Next-to-nearest neighbour couplings and entanglement generation in spin chains and optical arrays
10:45 – 11:00	*Break*
11:00 – 11:50	**Willard Miller, Jr.** Laplace equations, conformal superintegrability and Bôcher contractions
12:00 – 12:50	**Alexander Turbiner** Polynomial Integrable Systems
13:00 – 14:30	*Lunch break*
14:30 – 15:20	**Jiří Tolar** From graded contractions to Clifford groups of quantum computing
15:30 – 16:20	**Anatoly G. Nikitin** Superintegrable and supersymmetric systems with position dependent mass
16:30 – 16:45	*Break*
16:45 – 17:35	**Zuzana Masáková** On the spectra of Pisot-cyclotomic numbers
17:45 –	*Glass of wine*

Tuesday 31.5. Villa Lanna

9:30 – 10:00	**Veronique Hussin** General solutions of the supersymmetric $\mathbb{C}P^{N-1}$ sigma model and constant curvature surfaces
10:05 – 10:35	**A. Michel Grundland** On the Fokas-Gel'fand theorem for integrable systems
10:40 – 11:00	*Coffee break*
11:00 – 11:30	**Alexei V. Penskoi** Spectral geometry and symmetry reduction
11:35 – 12:05	**Ian Marquette** Family of N-dimensional superintegrable systems and quadratic algebra structures
12:10 – 14:15	*Lunch break*
14:15 – 14:45	**Ladislav Hlavatý** On uniqueness of T-duality with spectators
14:50 – 15:20	**Pavel Exner** Asymptotic expansions for singular Schrödinger operators and Robin billiards
15:25 – 15:45	*Break*
15:45 – 16:15	**Pavel Šťovíček** A family of explicitly diagonalizable weighted Hankel matrices generalizing the Hilbert matrix
16:20 – 16:50	**Antonín Hoskovec** Quantum State Transfer in one and two dimensions

Wednesday 1.6. Villa Lanna

9:30 – 10:00	**Hubert de Guise** Latin squares and mutually unbiased bases
10:05 – 10:35	**Edita Pelantová** On-line multiplication and division in real and complex bases
10:40 – 11:00	*Coffee break*
11:00 – 11:30	**Lenka Motlochová** On cubature rules associated to Weyl group orbit functions
11:35 – 12:05	**Agnieszka Tereszkiewicz** Three-Dimensional Hybrids With Mixed Boundary Value Problems
12:10 – 12:40	**Marzena Szajewska** Decomposition matrices for data sampled on the triangular lattices
19:00 –	*Conference dinner*

Thursday 2.6. Villa Lanna

9:30 – 10:00	**Yvan Saint-Aubin** Indecomposable representations in physics
10:05 – 10:35	**Piergiulio Tempesta** Formal groups and Statistical Mechanics
10:40 – 11:00	*Coffee break*
11:00 – 11:30	**Libor Šnobl** Integrable and superintegrable systems in static electromagnetic fields
11:35 – 12:05	**Sarah Post** Special Functions and the Smorodinsky–Winternitz Potentials
12:10 – 14:15	*Lunch break*
14:15 – 14:45	**Vladimir Dorodnitsyn** Invariant difference schemes for the Ermakov system
14:50 – 15:20	**Miguel A. Rodriguez** Construction of partial differential schemes : discrete variables, invariant lattices and the Schwarz theorem
15:25 – 15:45	*Break*
15:45 – 16:15	**Decio Levi** On Partial Differential and Difference Equations with Symmetries Depending on Arbitrary Functions
16:20 – 16:50	**Roman Kozlov** The adjoint equation method for constructing first integrals of difference equations

Friday 3.6. Villa Lanna

9:30 – 10:00	**Luigi Martina** Structure-preserving discretizations of nonlinear PDEs
10:05 – 10:35	**Mark A. Walton** Adjoint affine fusion and tadpoles
10:40 – 11:00	*Coffee break*
11:00 – 11:30	**Jiří Hrivnák** Weight- and Coweight-Lattice Discretization of Weyl-Orbit Functions
11:35 – 12:05	**Robert V. Moody** The 600 cell and a Discretization of $SU(2)$
12:10 – 12:40	Closing

List of abstracts

Hubert de Guise: *Latin squares and mutually unbiased bases*
Joint work with Mario Gaeta, Andrei Klimov and Olivia Di Matteo
Lakehead University

Latin squares have been studied as mathematical objects since Euler introduced the 36-officer problem in 1782. More recently, Fisher (amongst others) described how Latin squares can be used in the design of experiments; Latin squares have found a host of other applications.

Mutually unbiased bases (MUBs) have a much more recent history; Wootters showed they are an optimal choice of measurements to reconstruct the density matrix of a general quantum state. An example of MUBs are the generalized Pauli matrices $\wp$ introduced by Patera and Zassenhaus (in 1987) in their study of gradings.

In this presentation I will describe a connection between MUBs and orthogonal Latin squares. The common bridge linking the two are curves in discrete phase space: certain types of MUBs can be represented by a curve, and this curve can be used to generate - under the right conditions - a Latin square. Logical operations transforming sets of MUBs amongst themselves can transform one set of orthogonal Latin squares into another set.

Vladimir Dorodnitsyn: *Invariant difference schemes for the Ermakov system*
Joint work with E. Kaptsov
Keldysh Institute of Applied Mathematics, Russian Academy of Sciences

We consider a special case of the Ermakov system

$$\begin{aligned} \ddot{u} + \alpha u &= \tfrac{1}{u^2 v}, \\ \ddot{v} + \alpha v &= \tfrac{1}{v^2 u}, \end{aligned} \tag{1}$$

where $\alpha = const$. The symmetry of that system (named the Ermakov-Ray-Reid systems) possesses first integrals which can be computed with the help of the Noether theorem.

By means of difference analog of the Noether theorem we construct invariant schemes with difference first integrals. Developed schemes are as much integrable as the original differential equations.

[1] Ray, Reid - Ermakov Systems, nonlinear superositions and solutions of nonlinear equations of motion, J.Math.Phys, V. 21, 1980

[2] Dorodnitsyn V. Noether-type theorems for difference equations. Applied Numerical Mathematics, (2001)**39**, P.307–321.

[3] Dorodnitsyn V., Kozlov R., Winternitz P. Symmetries, Lagrangian formalism and integration of second order ODEs, J. of Nonlinear Math. Phys, v.10, N2, P.41-56, 2003.

[4] Dorodnitsyn V., Kozlov R., Winternitz P. Continuous symmetries of Lagrangians and exact solutions of discrete equations, J. Math.Phys. **45**(1) , P.336-359, 2004.

[5] Dorodnitsyn V., Kozlov R. Lagrangian and Hamiltonian formalism for discrete equations: symmetries and first integrals. SMS Lecture Notes, Cambridge University Press, P.7-49, 2011.

Pavel Exner: ***Asymptotic expansions for singular Schrödinger operators and Robin billiards***

Doppler Institute for Mathematical Physics and Applied Mathematics

The subject of this talk are spectral properties of several operator classes. They include Schrödinger operators with an attractive singular 'potential', supported by a manifold of a lower dimensionality. The simplest of them of them can be formally written as $-\Delta - \alpha\delta(x-\Gamma)$ with $\alpha > 0$, where Γ is a curve in $\mathbb{R}^d$, $d = 2, 3$, or a surface in $\mathbb{R}^3$; the expression can be modified to include a different singular interaction term or a regular potential bias. Another class are Hamiltonians describing quantum motion in a region with attractive Robin boundary. We discuss the ways in which spectral properties of such systems are influenced by the interaction support geometry, in particular, it the situation when the coupling constant is large, with an attention to similarities and differences between the operators considered.

A. Michel Grundland: ***On the Fokas-Gel'fand theorem for integrable systems***

Centre de Recherches Mathématiques & Department of Mathematics and Computer Science, Université du Québec

The Fokas-Gel'fand theorem on the immersion formula of 2D-surfaces is related to the study of Lie symmetries of an integrable system. A rigorous proof of this theorem is presented which may help to better understand the immersion formula of 2D-surfaces in Lie algebras. It is shown, that even under weaker conditions, the main results of this theorem is still valid. A connection is established between three different analytic descriptions for immersion functions of 2D-surfaces, corresponding to the following three types of symmetries: gauge symmetries of the linear spectral problem, conformal transformations in the spectral parameter and generalized symmetries of the integrable system. The theoretical results are applied to the $\mathbb{C}P^{N-1}$ sigma model and several soliton surfaces associated with these symmetries are constructed. It is shown that these surfaces are linked by gauge transformations.

Ladislav Hlavatý: ***On uniqueness of T-duality with spectators***
Joint work with Filip Petrásek
Department of Physics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague

We investigate the dependence of nonabelian T-duality on various identifications of the group of isometries with its orbits, i.e. on various locations of the group unit in manifolds invariant under isometry groups. We show that T-duals constructed by isometry groups of dimension less than the dimension of the (pseudo)riemannian manifold may depend not only on the initial metric but also on choice of manifolds defining positions of group units on each of the invariant submanifold. We investigate whether this dependence can be compensated by the coordinate transformation.

Antonín Hoskovec: ***Quantum State Transfer in one and two dimensions***
Joint work with Igor Jex
Department of Physics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague

Faithful placement of information into a specific position in the network (spin lattice) is one of the elementary tasks of Quantum Information Processing. This contribution will be an overview of our past endeavours in the field as well as of some current and future work. The focus of the contribution is twofold. First we will focus on the procedure known as Dynamical Decoupling and how it can be used to facilitate the Quantum State Transfer in different scenarios. Then the attention will shift towards our current research of factorization of Quantum State Transfer on a two dimensional spin lattice. The factorization is possible due to recurrence relations for eigenvalues of the Hamiltonians of the spin lattices.

Jiří Hrivnák: ***Weight- and Coweight-Lattice Discretization of Weyl-Orbit Functions***

Joint work with Mark Walton

Department of Physics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague

It is standard to perform the discrete Fourier analysis with Weyl-orbit functions on the coweight-lattice fragment contained in the fundamental region of the affine Weyl group. Here we compare the standard treatment to the recently-developed analysis on the fragment of the weight lattice. The weight-lattice discretization possesses symmetry between the labels and the arguments of the Weyl-orbit functions. This property allows the construction of unitary, symmetric matrices with Weyl-orbit-valued elements. For antisymmetric orbit functions, these matrices coincide with the Kac-Peterson modular S matrices. Consequences of the weight lattice discretization for the corresponding orthogonal polynomials are also discussed.

Veronique Hussin: ***General solutions of the supersymmetric*** $\mathbb{C}P^{N-1}$ ***sigma model and constant curvature surfaces***

Joint work with L. Delisle, İ. Yurduşen and W. J. Zakrzewski

Centre de Recherches Mathématiques & Département de mathématiques et de statistique, Université de Montréal

We present in this talk the general construction of finite action solutions of the supersymmetric $\mathbb{C}P^{N-1}$ sigma model. We start with a detailed study the supersymmetric $\mathbb{C}P^2$ model and give explicit holomorphic and non-holomorphic solutions. The results are thus generalised to $\mathbb{C}P^{N-1}$. Constant curvature surfaces are shown to be associated to an unique holomorphic solution, the generalised Veronese curve. In the non-holomorphic case, we give some set of such solutions. We also give some ways to extend our analysis to general supersymmetric Grassmannian models.

Roman Kozlov: ***The adjoint equation method for constructing first integrals of difference equations***

Joint work with V. Dorodnitsyn, E. Kaptsov and P. Winternitz

Department of Business and Management Science, Norwegian School of Economics

A new method for finding first integrals of discrete equations is presented. It can be used for discrete equations which do not possess a variational (Lagrangian or Hamiltonian) formulation. The method is based on a newly established identity which links symmetries of the underlying discrete equations, solutions of the discrete adjoint equations and first integrals. If a sufficient number of first integrals is found, it possible to provide the general solution of the discrete equations.

Decio Levi: ***On Partial Differential and Difference Equations with Symmetries Depending on Arbitrary Functions***
Dipartimento di Matematica e Fisica, Universita' degli Studi di Roma Tre

We present some ideas on when Lie symmetries, both point and generalized, can depend on arbitrary functions. We show a few examples, both in partial differential and partial difference equations where this happens. In most of the cases these symmetries appear when the systems are Darboux integrable. Moreover we show that the infinitesimal generators of generalized symmetries depending on arbitrary functions, both for continuous and discrete equations, effectively play the role of master symmetries.

Ian Marquette: ***Family of N-dimensional superintegrable systems and quadratic algebra structures***
School of Mathematics and Physics, The University of Queensland

Classical and quantum superintegrable systems have a long history and they possess more integrals of motion than degrees of freedom. They have many attractive properties, wide applications in modern physics and connection to many domains in pure and applied mathematics. We discuss two families of superintegrable Kepler-Coulomb systems with non-central terms and superintegrable Hamiltonians with double singular oscillators of type $(n, N-n)$ in N-dimensional Euclidean space. We present their quadratic and polynomial algebras involving Casimir operators of $so(N+1)$ Lie algebras that exhibit very interesting decompositions $Q(3) \oplus so(N-1)$, $Q(3) \oplus so(n) \oplus so(N-n)$ and the cubic Casimir operators. The realization of these algebras in terms of deformed oscillator enables the determination of a finite dimensional unitary representation. We present algebraic derivations of the degenerate energy spectra of these systems and relate them with the physical spectra obtained from the separation of variables.

Luigi Martina: ***Structure-preserving discretizations of non-linear PDEs***

Joint work with Decio Levi and Pavel Winternitz
Dipartimento di Mathematica e Fisica dell'Università del Salento, Sezione INFN di Lecce

Symmetry structures of partial differential equations can be reflected in difference schemes. In particular, the Liouville equation is a prototype of systems in which three different structure-preserving discretizations on four point lattices can be presented and, then, used to solve specific boundary value problems. The results are compared with exact solutions satisfying the same boundary conditions. One preserves linearizability of the equation, another the infinite-dimensional symmetry group as higher symmetries, the third preserves the maximal finite-dimensional subgroup of the symmetry group as point symmetries. A 9-point invariant scheme is also considered, but worse numerical solutions are presented and discussed.

Zuzana Masáková: ***On the spectra of Pisot-cyclotomic numbers***

Doppler Institute & Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague

The spectrum of a real number $\beta > 1$ is the set of polynomials with coefficients in a finite alphabet $\mathcal{A}$, evaluated in β, $X^{\mathcal{A}}(\beta) = \big\{ \sum_{j=0}^{n} a_j \beta^j : n \in \mathbb{N},\ a_j \in \mathcal{A} \big\}$. We consider the case when β is a Pisot-cyclotomic number of order n and the alphabet $\mathcal{A}$ of digits is taken to be the set of n-th roots of unity. Then $X^{\mathcal{A}}(\beta)$ is a discrete subset of cyclotomic integers possessing crystallographically forbidden symmetries. We focus on the cases where β is a quadratic or a cubic Pisot-cyclotomic number and compare the spectrum to the quasicrystal model obtained by the cut-and-project method.

Willard Miller, Jr.: ***Laplace equations, conformal superintegrability and Bôcher contractions***

Joint work with Ernest Kalnins, Eyal Subag and Adrian Escobar
University of Minnesota

Quantum superintegrable systems are solvable eigenvalue problems. Their solvability is due to symmetry, but the symmetry is often "hidden". The symmetry generators of 2nd order superintegrable systems in 2 dimensions close under commutation to define quadratic algebras, a generalization of Lie algebras. Distinct systems and their algebras are related by geometric limits, induced by generalized Inönü-Wigner Lie algebra contractions of the symmetry algebras of the underlying spaces. These have physical/geometric implications, such as the Askey scheme for hypergeometric orthogonal polynomials. The systems can be best understood by transforming them to Laplace conformally superintegrable systems and using ideas introduced in the 1894 thesis of Bôcher to study separable solutions of the wave equation. The contractions can be subsumed into contractions of the conformal algebra so(4,C) to itself. Here we announce main findings, with detailed classifications in papers submitted and under preparation.

Robert V. Moody: ***The* 600 *cell and a Discretization of*** $SU(2)$
Joint work with Jun Morita
University of Victoria

In 1998 Jiri Patera, Liang Chen, and I wrote a paper on non-crystallographic root systems, with particular emphasis on the icosahedral series, which exists in dimensions $2, 3, 4$. The model in dimension four is the 600-cell, a regular polyhedron with 600 faces and 120 vertices, with the latter being a non-cyrstallographic root system. Its symmetry group is the Coxeter group H_4 (of order 14400), which is generated by the reflections in these roots. In its standard coordinate representation the coefficients of the roots are all expressible in terms of integers along with the goldern ratio τ and its conjugate τ'. Curiously, simply conjugating all of these coefficients leads to a different set of coordinates, a different model of the root system, and a different version H_4' of the Coxeter group.

No one looks at the group H^∞ generated by these two Coxeter groups together because it is inifinte and the orbits of the roots of each type now lie densely on the sphere on which these roots lie. Nonetheless, we were interested in finding out more about what was happening. This paper looks at this group H^∞ and its corresponding 'root' system. This is more interesting and well-organized than it might first appear, and it leads to a sort of discreteization of $SU(2)$ and also an effective way of approximating elements of $SU(2)$ (and then $SO(3)$) by using products of a few simple matrices with coefficients of the form $(a + b\tau)/2$, where $a.b$ are integers, with n.

Lenka Motlochová: *On cubature rules associated to Weyl group orbit functions*

Joint work with Jiří Hrivnák and Lenka Háková

Department of Physics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague

The aim of this talk is to describe several cubature formulas related to the Weyl group orbit functions, i.e. to the special cases of the Jacobi polynomials associated to root systems. The link between the Weyl group orbit functions and the Jacobi polynomials is explicitly given. The four cubature rules corresponding to these polynomials are summarized for all simple Lie algebras and several numerical tests are shown. The additional Clenshaw-Curtis cubature formulas connected with the simple Lie algebra C_2 are presented.

Anatoly G. Nikitin: *Superintegrable and supersymmetric systems with position dependent mass*

Institute of Mathematics of National Academy of Sciences of Ukraine

Using the classical results of Patera and Winternitz concerning the subgroup structure of the fundamental groups of physics, first and second order integrals of motion for 2d and 3d quantum mechanical systems with position dependent masses are classified. The presented classification is preliminary but includes some new systems. In addition, supersymmetric aspects of the considered superintegrable systems are discussed and exploited to construct their exact solutions.

Edita Pelantová: *On-line multiplication and division in real and complex bases*

Joint work with Marta Brzicová, Christiane Frougny and Milena Svobodová

Doppler Institute & Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague

A positional numeration system is given by a base and by a set of digits. The base is a real or complex number β such that $|\beta| > 1$, and the digit set $\mathcal{A}$ is a finite set of real or complex digits (including 0).We first formulate a generalized version of the on-line algorithms for multiplication and division of Trivedi and Ercegovac for the cases that β is any real or complex number, and digits are real or complex. We show that if $(\beta, \mathcal{A})$ satisfies the so-called (OL) Property, then on-line multiplication and division are feasible by the Trivedi-Ercegovac algorithms. For a real base β and alphabet $\mathcal{A}$ of contiguous integers, the system

$(\beta, \mathcal{A})$ has the (OL) Property if $\#\mathcal{A} > |\beta|$. Provided that addition and subtraction are realizable in parallel in the system $(\beta, \mathcal{A})$, our on-line algorithms for multiplication and division have linear time complexity.

Alexei V. Penskoi: ***Spectral geometry and symmetry reduction***
Moscow State University

Spectral Geometry studies relationship between geometry of domains in an euclidean space or Riemannian manifolds and the spectrum of the Laplace operator on them. Classical questions of Spectral Geometry are well-known, among them one could mention "Can one hear the shape of a drum?" and "A drumhead of which shape produces the lowest possible sound among all drumheads of given area?" Natural problems of Spectral Geometry are very deep and their solution requires using Geometry, Topology, Analisys and Mathematical Physics including symmetry reduction.

Sarah Post: ***Special Functions and the Smorodinsky-Winternitz Potentials***
Department of Mathematics, University of Hawai'i at Mānoa

In this talk, I will discuss the special functions associated with the superintegrable potentials discovered by Winternitz and collaborators in the mid-60's and how they are still giving new insights into special functions and orthogonal polynomials. I will discuss the special functions associated with separation in various orthogonal coordinate systems and the inter-basis expansion coefficients as families of hypergeometric orthogonal polynomials, including their multi-variate extensions.

Miguel A. Rodriguez: ***Construction of partial differential schemes : discrete variables, invariant lattices and the Schwarz theorem***
Joint work with Decio Levi
Departamento de Física Téorica II, Facultad de Físicas, Universidad Complutense

Difference operators in several variables do not necessarily commute, their commutation depending on the lattice equations. I present in this talk a discussion on the conditions over a lattice to allow commutativity of these difference operators. I will also introduce a set of discrete variables allowing the construction of invariant schemes and apply the procedure to a simple example, the potential Burgers equation with two different lattices, an orthogonal lattice which is invariant under the symmetries of the equation and satisfies the commutativity of the partial difference operators and an exponential lattice which is not invariant and does not satisfy the Clairaut–Schwarz–Young theorem. I will discuss some preliminary results on the numerical approximations of two exact solutions of the potential Burgers equation using both types of lattices.

Yvan Saint-Aubin: ***Indecomposable representations in physics***
Département de mathématiques et de statistique, Université de Montréal and Centre de recherches mathématiques

Several models of statistical physics are described using non-semisimple algebras. For example the transfer matrices of percolation, dimers and the Ising model can be chosen to belong to the Temperley-Lieb algebra. These algebras have indecomposable representations that are physically relevant. (Recall that a representation is indecomposable if it is reducible, but not a direct sum of two proper subrepresentations.) These indecomposable representations are believed to be tied to the logarithmic conformal field theories and are thus physically important.

I shall give examples of such indecomposable representations in physical models and describe way to obtain complete lists of (isomorphism classes of) indecomposable representations.

Marzena Szajewska: ***Decomposition matrices for data sampled on the triangular lattices***
Joint work with Mark Bodner and Jiří Patera
Department of Mathematical Physics, Institute of Mathematics, University of Bialystok

We will discuss the method of exact calculation of Fourier decomposition in case when series of data on the same lattice have to be analyzed.

Libor Šnobl: ***Integrable and superintegrable systems in static electromagnetic fields***
Joint work with Antonella Marchesiello and Pavel Winternitz
Doppler Institute & Department of Physics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague

We consider a charged particle in three spatial dimensions moving in a static electromagnetic field described by the vector potential $\vec{A}(\vec{x})$ and the electrostatic potential $V(\vec{x})$. We look for potentials which allow sufficient number of independent integrals making the system integrable or even superintegrable. We concentrate on several choices of the assumed form of the integrals of the first or second order in momenta and study what are their implications for the structure of the potentials. Once we accomplish integrability by a suitable choice of potentials, we look for additional independent integrals of first or second order in the momenta which make the system superintegrable (minimally or maximally). We study their classical equations of motion and quantum spectra.

The talk is partly based on

A. Marchesiello, L. Šnobl and P. Winternitz, Three-dimensional superintegrable systems in a static electromagnetic field, J. Phys. A: Math. Theor. 48 (2015) 395206, IOPSelect & J. Phys. A Highlights of 2015.

Pavel Šťovíček: ***A family of explicitly diagonalizable weighted Hankel matrices generalizing the Hilbert matrix***
Doppler Institute & Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague

A three-parameter family $B = B(a, b, c)$ of weighted Hankel matrices is introduced with the entries

$$B_{j,k} = \frac{\Gamma(j+k+a)}{\Gamma(j+k+b+c)} \sqrt{\frac{\Gamma(j+b)\Gamma(j+c)\Gamma(k+b)\Gamma(k+c)}{\Gamma(j+a)\, j!\, \Gamma(k+a)\, k!}}\,,$$

$j, k \in \mathbb{Z}_+$, assuming that a, b, c are positive and $a < b + c$, $b < a + c$, $c < a + b$. The famous Hilbert matrix given by

$$B_{j,k} = \frac{1}{j+k+\theta}$$

is included as a particular case for $a = b = \theta$, $c = 0$. Symmetry properties of the matrix B are revealed. The symmetry can be used to construct an explicit diagonalization of the matrix while heavily employing the theory of orthogonal polynomials. This contribution is based on the paper

T. Kalvoda, P. Šťovíček: A family of explicitly diagonalizable weighted Hankel matrices generalizing the Hilbert matrix, Linear Multilinear Alg. **64** (2016) 870-884.

Piergiulio Tempesta: ***Formal groups and Statistical Mechanics***

Departamento de Física Téorica II, Facultad de Físicas, Universidad Complutense

We will show that an intrinsic group-theoretical structure is at the heart of the notion of entropy. This structure emerges when imposing the requirement of composability of an entropy with respect to the union of two statistically independent systems. A new formulation of the celebrated Shannon-Khinchin set of axioms is proposed, obtained by replacing the additivity axiom with that of composability.

The theory of formal groups offers a natural language for our group-theoretical approach to generalized entropies. In this settings, the known entropies can be encoded into a general trace-form class, the universal-group entropy (so called due to its relation with the Lazard universal formal group of algebraic topology).

We shall also prove that Renyi's entropy is the first example of a new family of non trace-form entropies, of potential interest in the theory of complex systems, called the Z-entropies. Each of them is composable and, in particular, generalizes simultaneously the entropies of Boltzmann and Renyi (obtained under suitable limits). The information theoretical content of composable entropies is shown to be a byproduct of their underlying group structure.

The theory of group entropies will also be related with a new construction in analytic number theory of L-series and generalized Bernoulli polynomials.

Agnieszka Tereszkiewicz: ***Three-Dimensional Hybrids With Mixed Boundary Value Problems***

Joint work with Marzena Szajewska

Department of Differential Equations, Institute of Mathematics, University of Bialystok

Boundary value problems are considered on a simplex F in the real Euclidean space $\mathbb{R}^3$. The recent discovery of new families of special functions, orthogonal on F, makes it possible to consider not only the Dirichlet or Neumann boundary value problems on F, but also the mixed boundary value problem which is a mixture of Dirichlet and Neumann type, ie. on some parts of the boundary of F a Dirichlet condition is fulfilled and on the other Neumann's works.

Jiří Tolar: ***From graded contractions to Clifford groups of quantum computing***

Doppler Institute and Department of Physics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague

Among the gradings of simple Lie algebras over complex numbers the most important ones are the gradings by the maximal torus, also called root or Cartan decomposition. Such a grading means a decomposition into eigenspaces of the maximal torus.The question about the existence of other fine gradings has been raised in 1989 in the seminal paper by J. Patera and H. Zassenhaus and systematically solved for the simple Lie algebras over complex and also real numbers by the research group collaborating with J. Patera in Prague (M. Havlíček, E. Pelantová, M. Svobodová, J. Tolar). The gradings have then been used for constructing of grading preserving contractions (graded contractions) of semisimple Lie algebras. Since the system of quadratic equations for contraction parameters often gets quite large, its solution is simplified by the knowledge of symmetries formed by the automorphisms which leave the given grading invariant, as shown in papers by J. Hrivnák, P. Novotný, J. Patera and J. Tolar. In quantum computing, the Clifford groups were introduced in 1998 in the context of quantum stabilizer codes by D. Gottesman as the groups of unitary operators generating symmetries of the Pauli grading.

Alexander Turbiner: ***Polynomial Integrable Systems***

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México

Notion of a *polynomial integrable system* is introduced. It is stated that (i) any Calogero-Moser model (including TTW model) is canonically-equivalent to a polynomial integrable system, its Hamiltonian and integrals are polynomials in p and q.
(ii) for any Calogero-Moser model there exists a change of variables in which the potential in a rational function.
(iii) any Calogero-Moser model is equivalent to Euler-Arnold top in a constant magnetic field with algebra gl(n) (for a classical Weyl group) with constancy of Casimir operators as a constraint. (iv) 3-body elliptic Calogero is presented as an example.

Luc Vinet : *Next-to-nearest neighbour couplings and entanglement generation in spin chains and optical arrays*

Joint work with Matthias Christandl and Alexei Zhedanov
Centre de recherches mathématiques & Département de physique, Université de Montréal

It is known that perfect state transfer (PST) can be achieved in XX spin chains with properly engineered nearest-neighbour (NN) couplings. The simplest model with this feature is based on the Krawtchouk polynomials. In view of the mathematical equivalence between the equations governing the dynamics of single excitations in XX spin chains and those describing photon propagation in arrays of evanescently coupled waveguides, PST can be experimentally realized in photonic lattices. In this context restricting to NN interactions is obviously an approximation.

The phenomenon of fractional revival (FR) or wave packet splitting that has smaller but identical packets reproduce periodically, can also be seen in certain XX spin chains. This is not so however in the NN Krawtchouk model. Like PST, FR brings useful new tools in quantum information and can generate for instance quantum entanglement.

I shall present an analytic extension of the NN Krawtchouk model that includes next-to-nearest neighbour couplings. Under certain conditions, it will be shown to admit PST as well as FR in distinction to the NN situation. Its application to coherent transport in photonic lattices will be discussed.

Mark A. Walton: *Adjoint affine fusion and tadpoles*

Joint work with Andrew Urichuk
Department of Physics and Astronomy, University of Lethbridge

Elementary and universal formulas will be derived for affine fusion with the adjoint representation, using the (refined) affine depth rule. We show that off-diagonal adjoint affine fusion is obtained from the corresponding tensor product by simply dropping non-dominant representations. For diagonal fusion, a coefficient equals the number of nonzero Dynkin labels of the relevant affine highest weight, minus 1. This is confirmed easily in the phase-model description of affine fusion. A nice lattice-polytope interpretation follows, and allows the straightforward calculation of the genus-1 1-point adjoint Verlinde dimension, the adjoint affine fusion tadpole. We show that explicit formulas, (piecewise) polynomial in the level, can be written.

List of participants

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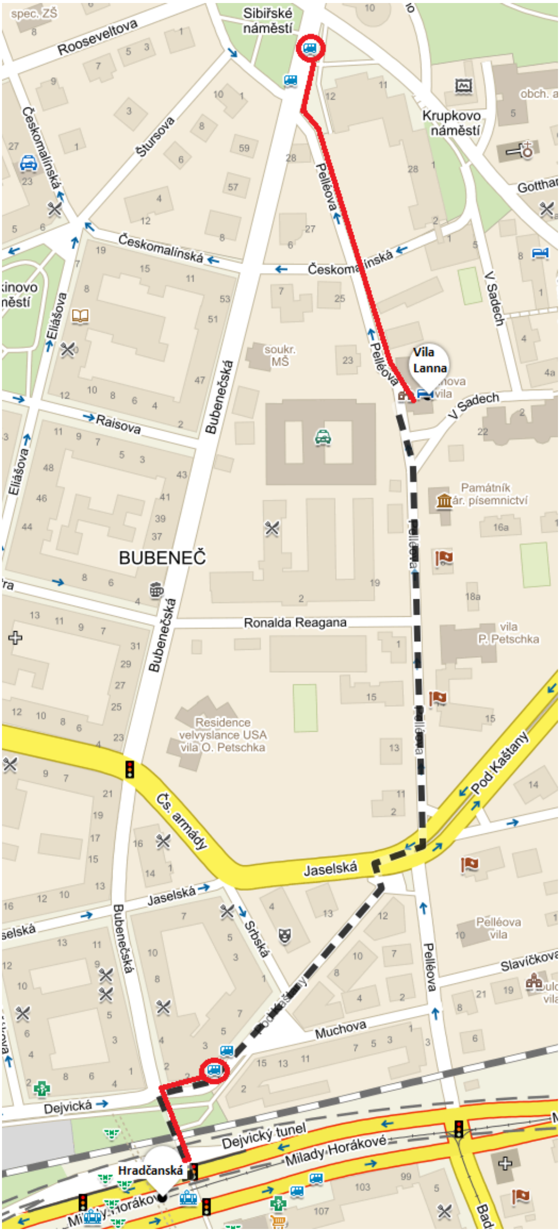
From Hradčanská metro station to Villa Lana



- - - from Hradčanská to Villa Lanna by walk (680m) or alternatively
—— by Bus 131 to Sibiřské Náměstí and walk (260m) to Villa Lanna

From Staroměstská metro station to CTU

- - - from Staroměstská to CTU by walk (330m) or alternatively
—— one station by tram number 17 to Právnická Fakulta walk (100m) to CTU

Schedule

CTU B103	**Monday**
9:30 - 9:45	*Opening*
9:45 -10:35	Vinet
10:45-11:00	*Break*
11:00-11:50	Miller
12:00-12:50	Turbiner
13:00-14:30	*Lunch break*
14:30-15:20	Tolar
15:30-16:20	Nikitin
16:30-16:45	*Break*
16:45-17:35	Masáková
17:45-	*Glass of wine*

Villa Lanna	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
9:30 -10:00	Hussin	de Guise	Saint-Aubin	Martina
10:05-10:35	Grundland	Pelantová	Tempesta	Walton
10:40-11:00	*Coffee break*			
11:00-11:30	Penskoi	Motlochová	Šnobl	Hrivnák
11:35-12:05	Marquette	Tereskiewicz	Post	Moody
12:10-12:40		Szajewska		*Closing*
	Lunch break			
14:15-14:45	Hlavatý		Dorodnitsyn	
14:50-15:20	Exner		Rodriguez	
15:25-15:45	*Break*		*Break*	
15:45-16:15	Šťovíček		Levi	
16:20-16:50	Hoskovec		Kozlov	
19:00-		*Conference dinner*		